On the Investment Network and Development*

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Abstract

Capital accumulation and the systematic reallocation of economic activities across sectors are two of the most salient features of the process of economic development. These two processes are interconnected through the production of capital of various types and the intensity of use of different capital across sectors. All this information is summarized by the investment network. Our paper introduces the first harmonized measures of the investment network across countries at different stages of development, and proposes a theory that characterizes disparities in income per capita (and growth rates) across countries based on these connections. Through counterfactual exercises, we show that 31% of disparities in income per capita can be accounted for by disparities in the investment network. This role is mediated by the intensity of capital use in these economies and its interaction with the IO structure. We show that imposing the investment network of an advance economy to all countries may be detrimental to output per worker in certain economies. This result suggests that observed disparities investment bundles may reflect optimal responses to domestic sectorial productivities and the economy's stage of development.

JEL Codes: E23; E21; O41.
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1 Introduction

Capital accumulation and the systematic reallocation of economic activities across sectors are two of the most salient features of the process of economic development. Sectors utilize different investment goods for production. As economic activity shifts across sectors, the economy's ability to produce new capital—or to export goods in exchange for these goods—changes. This facilitates further capital accumulation of various types and additional sectorial reallocation. Studying the nature of this continuous feedback is crucial for understanding the mechanics of economic development and it requires measures of sectorial links in both the production and use of new capital, i.e. investment networks. This paper provides the first harmonized measures of investment networks across countries and at different stages of development; documents novel facts about how this network evolves as countries develop; and proposes a theory to evaluate its impact on the observed income differences across countries.

The importance of the nature of the investment network for economic development can be traced back to Hirschman (1958). He argued that a successful development strategy, along with the corresponding paths of capital accumulation, should emphasize sectors with strong forward and backward linkages in the production of new capital. Our exercise formalizes these ideas while bringing empirical content to it. Recent studies have highlighted the changing sectorial composition of the production of (aggregate) investment in the economy Garcia-Santana, Pijoan-Mas and Villacorta (2021); Herrendorf, Rogerson and Valentinyi (2021), as well as disparities in the bundles of capital goods used for production Caunedo and Keller (2023). We propose a theory that rationalizes the full structure of production and uses of different capital types along the development spectrum. We then characterize the elasticity of aggregate output to changes in sectorial TFP as function of the investment network and its interaction with another important source of linkages in the production, the input-output structure. We show that these elasticities are non-linear functions of the nature of these networks, and that importantly, the role of a sector in boosting economic activity depends its importance in the production of new investment, its relevance in the production of value added and the capital intensity of other sectors in the economy. Thus, while Hirschman's hypothesis about the role of investment linkages was correct, it was incomplete.

We use our theory to inform measurement. We call the vector that summarizes aggregate output elasticities to sectorial TFP the "influence vector", following the now extensive literature that studies network properties of the economy, Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012). The influence vector summarizes the direct and indirect impact of changes in sectorial productivity and in the terms of trade for aggregate economic activity. Influence is a function of the input-output structure as well as of the investment network of the economy, through an augmented Leontief inverse. While measures of the input-output structure have become increasingly available across countries, estimates of the investment network are only available for the US (vom Lehn and Winberry, 2022) and a handful of advanced economies

(Ding, 2023).[1] We advance previous measurement efforts by providing harmonized estimates of the investment network across 22 countries at different stages of development, i.e. incomes per capita between $1,450 and $112,229 constant PPP dollars. For many countries, noticeably South Korea, we provide time-series estimates of the investment network that go back to 1960s. In our analysis, capital is disaggregated into multiple equipment types, including ICT, Electronics, Machinery and Transportation; as well as structures, measured through Construction investment.[2]

To create our new harmonized measures of the investment network, we exploit a methodology similar to that of the Bureau of Economic Analysis (BEA) in the US. The BEA combines the occupational composition of each industry and an allocation rule for capital to workers, to estimate investment by capital type and sector. Unfortunately, the apportioning of stocks to workers is not publicly available. Hence, to assure replicability, we opt for an allocation of capital across sectors that follows Caunedo, Jaume and Keller (2023) for equipment; and an allocation that follows intermediate inputs for construction and other sectors with positive investment in final uses. While the allocation of investment may seem arbitrary, it is reassuring that our own estimates of capital-flow tables in the US follow closely those published by the BEA.[3]

We start by documenting systematic disparities in homophily between the IO and the investment network, which leads to differential roles for sectors as producers of (new) capital and intermediate goods for others. In other words, the diagonal of the IO structure is heavier than that of the investment network. Hence, while the investment and the input-output networks are both sources of amplification of productivity shocks, their empirical nature is different, and warrant differential impact on aggregates.

A useful summary statistic to measure the relevance of sectors as providers of investment is the the *outdegree* of a sector in the network, which corresponds to the row-sum of the entries in the network. Thus, outdegrees measure forward linkages which are strongly related to "upstreamness", as defined by Antras, Chor, Fally and Hillberry (2012). On average, countries' investment diversifies as countries develop. In poorer economies such as Ghana and Ethiopia, the sectors with highest out-degrees are Construction and Services. In richer economies, ICT, Construction and Transportation report the highest outdegrees, while the levels of outdegrees become more similar across sectors. So are sectors with high *outdegrees* in the investment network also sectors where changes in productivity have the strongest impact on aggregate activity? The answer is no. Our theory predicts that *influence* is the correct metric to answer this question.

We construct measures of influence using country-year variation in our estimates of the investment network, paired with measures of sector and country-specifc sectorial capital shares,

[1] These measures are self-reported by country offices to the OECD Statistics office, and it is unclear whether measurement is comparable across countries.

[2] Our benchmark estimates include 8 sectors but estimates for 19 sectors can be made readily available.

[3] The mean square error in the network between 1960 and 2014 is 0.04.

measures of the IO structure, and estimates of sectorial expenditure shares in value added. We then study the role of the investment network for persistent differences in income per capita through accounting exercises. We calibrate cross-country sectorial productivity differences to match disparities in value added across sectors within countries, and across countries for a sector. This way, our model matches exactly the variance in output per capita observed in the data. We then study the role of different channels in driving those disparities by first eliminating the investment network altogether, then eliminating the IO structure altogether. Our main finding is that the investment network accounts for 31% of the observed disparities in income per worker, and that the IO structure can account for 66% of the observed differences. A lower amplification effect from disparities in the investment network is not surprising given higher sectorial gross output elasticities to intermediate inputs than to capital: the former is 0.58 on average while the latter is 0.10 on average across sectors and countries.

Disparities in the investment network reflect differences in the technology used for production, possibly as a consequence of distortions that shift relative prices or as a consequence of disparities in comparative advantage. We explore the role of systematic disparities in the network along the development spectrum through counterfactuals were we replace the observed investment network by the investment network in the US in 1965 (i.e. a dated form of production) and in 2014 (i.e. a modern form of production). We show that poorer economies benefit relatively more from producing with the investment network of the US in 1965 (given their IO structure, sectorial productivities and patterns of final expenditure shares), while richer economies would benefit relatively more from producing with the investment network of the US in 2014. A "dated" investment network is particularly detrimental to richer economies. We find that imposing the investment network of an advanced economy to all countries may be detrimental to output per worker in certain economies, albeit it shrinks overall income disparities.

Contribution to the literature. There is a growing literature studying the relevance of sectorial linkages for differences in income per capita across countries. The role of intermediate input linkages has been highlighted by Ciccone (2002); Jones (2011). This role has been quantified in Fadinger, Ghiglino and Teteryatnikova (2022), who employs cross-country measures of input-output linkages as measured from the World Input-Output Dataset (WIOD) to show that differences in the IO structure across countries amplify the role of sectorial TFP for differences in income per capita. We show that the IO and the investment networks are empirically different, leading to different conclusions in terms of the role of sectors in boosting aggregate economic activity. In addition, the investment network directly affects the rate of convergence in the economy to its balanced growth path, making it an interesting object of study on its own right.

We contribute to the literature by allowing for dynamics in the accumulation of capital across sectors. Unitary elasticities of substitution in sectorial investment aggregators, as well as in intermediate inputs allow us to handle the empirical heterogeneity in factor intensities across sectors and the dynamics of capital-accumulation, while being consistent with balanced growth. The economy is efficiency, so when assessing welfare, Domar aggregation holds. That is, aggregate welfare is a weighted sum of productivity growth, weighted by Domar weights. As in Acemoglu *et al.* (2012) and Liu (2019) output elasticities to sectorial productivity are different than Domar weights. In our framework, this is the result of non-trivial dynamics in capital rather than distortions in production. A static version of our economy with full capital depreciation eliminates this disparity.

The main empirical contribution of our paper is to construct harmonized estimates of the investment network for many countries across time. We believe this effort opens the door to studying a myriad of questions related to the link between structural transformation and investment, already hinted in Garcia-Santana *et al.*, 2021; Buera, Kaboski, Mestieri and O'Connor, 2020, as well as further studies of the nature and timing of investment in particular goods to the overall path of development. Buera and Trachter (2024) is a good example in this direction within a frictional environment.

Highlighting the role of imported equipment for economic growth brings new relevance to the higher cost of investment relative to consumption in poorer countries, and these country's ability to generate resources to trade for these capital goods, Hsieh and Klenow (2007). Gaggl, Gorry and vom Lehn (2023) and Foerster, Hornstein, Sarte and Watson (2022) study the properties of the investment network in the US within a closed economy framework. Foerster *et al.* (2022) abstract from feedback effects between imported capital, the stock of capital available in the economy and sectorial output by assuming that either the share of imported investment is small, or that there are no time-trends in the terms of trade. Neither of these assumptions is realistic for the economies that we study. Gaggl *et al.* (2023) run their quantitative analysis with a single capital good for production, i.e. investment aggregators are assumed identical.

2 A model of the investment network and economic development

We build a framework to study the impact of long-term shifts in the composition of imported investment across sectors, as well as TFP growth in sectors producing equipment and structures for aggregate GDP growth.we do this in a context where markets for input and output are complete, and therefore we can characterize allocations through the technologies available to a planner.

The economy consists of N sectors that combine capital, labor and intermediate inputs to produce output.

$$y_{nt} = \left(\frac{v_{nt}}{\gamma_{nt}}\right)^{\gamma_{nt}} \left(\frac{m_{nt}}{1-\gamma_{nt}}\right)^{1-\gamma_{nt}}, \qquad \text{for } \gamma_{nt} \in [0,1],$$

a measure of value added $v_{nt} = \exp(z_{nt}) \left(\frac{k_{nt}}{\alpha_n}\right)^{\alpha_n} \left(\frac{l_{nt}}{1-\alpha_n}\right)^{1-\alpha_n}$ that depends on productivity z_{nt}, and capital and labor allocations, k_{nt}, l_{nt}; and a constant returns to scale intermediate input aggregator $m_{nt} = \prod_{i=1}^{N} \left(\frac{m_{int}}{\mu_{int}}\right)^{\mu_{int}}$ with $\sum_i \mu_{int} = 1$. The amount of intermediate inputs from sector i used in sector n is m_{int}. This flow of intermediate inputs is summarized by an IO matrix, M_t, with typical element μ_{int}. The rows of M_t add to the importance of a sector as an intermediate inputs provider to the rest of the economy, the columns summarize the input composition of the intermediate input bundle in a sector. It will also be convenient to define $\Gamma_t = \text{diag}\{\gamma_{nt}\}$, a matrix of value added shares in production, as well as matrix of capital expenditure shares, $\alpha_t = \text{diag}\{\alpha_{nt}\}$.

The capital stock used in each sector evolves according to the following law of motion,

$$k_{nt+1} = x_{nt+1} + (1 - \delta_n)k_{nt},$$

for a composite of investment from different sectors.[4]

We populate the economy by a continuum of firms that produce investment goods for each sector. Thus, the shape of the investment aggregator, as summarized by the intensity of use of different equipment types ω_{int}, is optimally chosen given a menu of technologies available in each economy at a point in time. These firms maximize profits by choosing the amount of investment in each equipment type, but also the intensity of use of each equipment for production,

$$\max_{\omega_{int}, \chi_{int}} r_{nt}x_{nt} - \sum_i p_{it}\chi_{int}$$

subject to

$$x_{nt} = \prod_{i=1}^{N} \left(\frac{\chi_{int}}{\omega_{int}}\right)^{\omega_{int}}, \tag{1}$$

$$\sum_i \xi_{int}\omega_{int}^{\nu_n} = B_n \tag{2}$$

for $\sum_{i=1}^{N} \xi_{int}^{\frac{1}{1-\nu_n}} = 1$, and ω_{int} the expenditure share in investment from sector n in sector j. We assume $\nu_n > 1$ which assures an interior solution to the technology choice problem. The technology frontier, is a generalization of Caselli and Coleman (2006) to allow for multiple equipment types, and sector-specific technology barriers, B_n. A key difference to their environment is that firms choose expenses in goods that are produced within the economy, rather than endowment goods.[5] Hence, there is a potentially a non-trivial feedback between the nature of the investment network, IO structure, and expenditure shares, which determine relative prices and technology choice.

Finally, inputs from sector i into the production of investment in other sectors, χ_{it} can be domestically produced or imported, $\chi_{int} = (\frac{\chi_{int}^d}{1-\phi_i})^{1-\phi_i}(\frac{\chi_{int}^f}{\phi_i})^{\phi_i}$, where ϕ_i is the expenditure share in foreign inputs for capital type i.

Each sectors' output can be used for production of final goods, c, intermediate uses m, or domestic investment, χ^d. Final good uses include domestic consumption and exports. Hence,

[4] A timing of investment that is contemporaneous to the stock that is being used in the period simplifies the notations without substantial changes to the analysis, because we focus on steady state allocations.

[5] In addition, we set the elasticity of substitution across goods to 1, $1/(1 - \sigma_n)$ for $\sigma_n = 0$, but results go through if we allow for an arbitrary elasticity of substitution such that $\nu_n > 1/(1 - \sigma_n)$.

feasibility requires

$$y_{nt} = c_{nt} + \sum_j m_{njt} + \sum_j \chi^d_{njt}.$$

The sectorial output allocated to the final good is combined with a homothetic aggregator,

$$Y_t = \prod_{n=1}^{N} \left(\frac{c_{nt}}{\theta_n}\right)^{\theta_n}, \qquad \sum_{n=1}^{N} \theta_n = 1 \text{ and } \quad \theta_n > 0.$$[6]

The final good can be used for consumption of the representative household, who derives utility $U(C_t)$, or for exports, $\boldsymbol{\epsilon}$.[7] Preferences satisfy usual regularity conditions.

$$Y_t = C_t + \epsilon_t.$$

We need to define the value of next exports in the economy as the difference in the value of exports and imports

$$NX_t = \epsilon_t - p_{\epsilon^f t}\epsilon^f_t.$$

The value of imports is the product between the price index of imports and a composite import value $\epsilon^f_t = \prod_{i=1}^{N} \frac{\chi^f_{it}}{\psi^f_{it}}^{\psi^f_{it}}$, as in Basu, Fernald, Fisher and Kimball (2005).[8] Hence, the terms of trade in this economy are given by the ratio between the price of final output (and therefore of exports) and the price of imports $\tau \equiv \frac{1}{p_{\epsilon^f t}}$. The price of imported goods is exogenous to the economy, and assuming trade balance pins down the value of exports in equilibrium. The price of imported goods is a CRS aggregator of the (exogenous) prices of imported investment for production.

2.1 Technology Choice

Optimality in the choice of technologies requires,

$$\frac{\omega_{jnt}}{\omega_{int}} = \left(\frac{\xi_{jnt}}{\xi_{int}}\right)^{\frac{1}{1-\nu_n}},$$

and a relative demand for investment goods that follows the relative intensities. [9]

[6]The model could be readily extended to allow for non-homotheticicities in this aggregator and therefore non-trivial income effects, as well as a homothetic aggregator with arbitrary elasticity of substitution. Then, the analysis that follows should be conducted along a constant growth path where the interest rate is constant but output shares in different sectors are changing, as in models of structural change.

[7]We could accommodate two different aggregators, one for exports, and one for domestic consumption. Results carry through except that the price of consumption should be defined in units of exports. This choice maintains the interpretation of the terms of trade that we describe below.

[8]In general, any unitary elasticity aggregator of exports and imports would preserve the balanced growth path properties discussed in the Appendix.

[9]It is simple to show that

$$\frac{\chi_{int}}{\chi_{jnt}} = \left(\frac{\xi_{jnt}}{\xi_{int}}\right)^{\frac{1}{1-\nu_n}} \left(\frac{p_{jt}}{p_{it}}\right)$$

Hence, the optimal (relative) intensity of use of each equipment category reflects the shape of the production possibility frontier, and through it, the menu of technologies available in each country for a given sector.

The level of the intensity is pin down by the height of the productivity possibility frontier, given a normalization of the shape parameters, i.e. $\xi_{jnt} = 1$.

$$\omega_{jnt} = B_n^{\frac{1}{\nu_n}}.$$

2.2 Equilibrium characterization. Closed economy.

We first study a closed economy. This amounts to assuming $\phi_i = 0$, or $\chi^f_{it} = 0$ in all sectors i, and no exports $\epsilon_t = 0$.

Expenditure shares in the investment aggregators are simply the parameters characterizing the shape of the production possibility frontier in each economy $\omega_{int} \propto \xi_{int}^{\frac{1}{1-\nu_n}}$, and independent of relative prices.

Let the Domar weight of sector n be $\eta_n \equiv \frac{p_n y_n}{p\nu}$, let the share of value added allocated to the production of final goods be $\zeta_n \equiv \frac{p_n c_n}{p\nu}$ and the value added share of each sector be $\tilde{\zeta}_n \equiv \zeta_n + \frac{p_n \chi^d_n}{p\nu}$.[10] Let Γ collect the vector of value added shares in production, and α collect capital intensities across sectors.

Proposition .1. *The equilibrium Domar weights are functions of sectorial investment rate.*

$$\left[I - \Gamma\alpha\Omega\frac{\mathbf{x}}{\mathbf{k}} - (1-\Gamma)M\right]^{-1}\boldsymbol{\zeta} \equiv \boldsymbol{\eta} \tag{3}$$

or in vector form

$$\eta_n = \zeta_n + \sum_{i=1}^{N} \alpha_i \gamma_i \omega_{ni} \frac{x_i}{k_i} \eta_i + \sum_{j=1}^{N} (1-\gamma_j)\mu_{nj}\eta_j.$$

Investment rates affect the level of Domar weights in our economy due to the dynamic nature of capital accumulation. This channel is muted in static models of intermediate input trade, or where the investment network is treated as a succeccion of static economies.

If the economy displays full depreciation $\delta_n = 1$, then $\frac{\mathbf{x}}{\mathbf{k}} = \mathbf{1}$ and the equilibrium Domar weights are independent of the investment rates. Alternatively, if depreciation is partial and the economy is in steady state, the investment rate is constant and a function of the rate of economic obsolescence, i.e. physical depreciation plus investment specific technological change, as we show the Appendix. Along the transition path, this economy displays interesting interactions between the investment rates and the Domar weights, which affect the pass through between productivity and aggregate value added, as we show next.

Proposition .2. *The equilibrium level of value added in the economy satisfies*

$$\ln(\nu) = \tilde{\boldsymbol{\eta}}'\Gamma\boldsymbol{\lambda} - \tilde{\boldsymbol{\eta}}'\Gamma(1-\alpha)\ln(\Gamma(1-\alpha)\boldsymbol{\eta})$$

[10] The share of value added allocated to final good production includes a constant expenditure share from the final output aggregator and the share of the final good in aggregate value added, $\zeta_n = \theta_n \frac{Y}{p\nu}$.

for a vector of sectorial influence $\tilde{\boldsymbol{\eta}} \equiv \tilde{\boldsymbol{\zeta}}'\Xi$, the product between sectorial value added shares, $\tilde{\boldsymbol{\zeta}}'$, and an adjusted Leontief inverse $\Xi \equiv (I - \Gamma\alpha\Omega - (1-\Gamma)M)^{-1}$. Value added is therefore a function of sectorial productivities, $\boldsymbol{\lambda}$, and a constant that is a function of investment rates through equilibrium Domar weights, $\boldsymbol{\eta}$.

In vector form

$$\ln(\nu) = \sum_n \tilde{\eta}_n \gamma_n z_n - \ln(\sum_n \gamma_n (1-\alpha_n)\eta_n) \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n).$$

The first two terms are common to factor models of the input-output structure of the economy, where the impact of total factor productivity depends on the input-output structure. The first term showcases the impact of productivity on value added and depends on the value added shares in production through Γ, and the vector of sectorial influence $\tilde{\eta}$, similarly to Acemoglu *et al.* (2012). Distinctively from Liu (2019), a wedge between influence and Domar weights occurs even absent distortions in the economy, and due to the presence of investment. The second term showcases the impact of disparities in the employment allocation across sectors. The entire sectorial employment distribution matters for value added as a consequence of the disparities in capital-labor ratios across sectors.[11]

Domar weights are a measure of the sectorial size as summarized in the total value of production resources (i.e. the value of gross output) relative to total value added. They can be rewritten as function of sectorial value added shares (instead of consumption shares) so that they are more easily comparable to the influence vector. Then, $\boldsymbol{\eta} = \tilde{\boldsymbol{\zeta}}(I - (I-\Gamma)M)^{-1}$, and one concludes that influence is always larger than the Domar weight of the sector whenever there is a non-trivial investment network. In other words, increases in sectorial productivity augment value added through input-output linkages and investment-network linkages. Influence is the correct sectorial weight for income and growth accounting.

2.3 Equilibrium characterization. Open economy.

We are now ready to extend the previous two results to an economy where final goods and investment are tradable, as in the benchmark.

Proposition .3. *The equilibrium Domar weights are functions of sectorial investment rate.*

$$\left[I - \Gamma\boldsymbol{\alpha}(1-\boldsymbol{\phi})\Omega\frac{\mathbf{x}}{\mathbf{k}} - (1-\Gamma)M\right]^{-1}\boldsymbol{\zeta} \equiv \boldsymbol{\eta} \tag{4}$$

or in vector form

$$\eta_n = \zeta_n + \sum_{i=1}^{N} \alpha_i \gamma_i \omega_{ni}(1-\phi_i)\frac{x_i}{k_i}\eta_i + \sum_{j=1}^{N}(1-\gamma_j)\mu_{nj}\eta_j.$$

Hence, the main difference to the closed economy version is that the investment network term is scaled by the importance of domestic investment, $(1-\phi) \in (0,1)$. The lower the

[11] In the current framework this is a consequence of heterogeneity in production technologies, but one could envision this feature being the consequence of distortions in factor prices across sectors or other policies.

importance of domestic investment, the closer the Domar weight is to the standard expression in a an economy with only intermediate input trade.

So is it imported capital important at all? The answer is yes, and to understand its role we turn to the expression for value added in the economy.

Proposition .4. *The equilibrium level of value added in the economy satisfies*

$$\ln(\nu) = \left(I - \Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}'\right)^{-1}\left(\tilde{\boldsymbol{\eta}}'\Gamma(\boldsymbol{\lambda} + \alpha\boldsymbol{\phi}\Omega'\boldsymbol{\tau}) - \tilde{\boldsymbol{\eta}}'\Gamma(1-\alpha)\ln(\Gamma(1-\alpha)\boldsymbol{\eta})\right)$$

or in vector form

$$\begin{aligned}\ln(\nu)(1 - \sum_n \gamma_n\alpha_n \sum_j \omega_{jn}\phi_j) &= \sum_n \tilde{\eta}_n\gamma_n z_n - \ln(\sum_n \gamma_n(1-\alpha_n)\eta_n)\sum_n \tilde{\eta}_n\gamma_n(1-\alpha_n) + \\ &\quad \sum_n \tilde{\eta}_n\gamma_n\alpha_n \sum_j \omega_{jn}\phi_j \ln(\tau_j).\end{aligned}$$

First, the presence of tradable investment goods induces and additional amplification (as in Jones (2011) for tradable intermediate inputs). The reason is that as productivity increases within the economy, the export capacity improves, and due to trade balance that implies higher imports of investment. The strongest the dependence on imported equipment and the intensity of use of capital, the strongest this amplification channel is. Second, the terms of trade enter as a channel directly affecting value added in the economy. Once adjusted for the role of imported investment, through the capital share and the investment network), the terms of trade affect the economy similarly to a TFP shock.

Notice that as $\phi \to 0$ the economy losses is dependence in tradable investment, and Proposition .3 and .4 boil down to their closed economy versions.

To ease the exposition, we discuss the characteristics of the BGP in Appendix A.1.1. The presence of unitary elasticity investment aggregators, as well as sectorial production technologies assures the existence of a BGP with heterogeneous rates of technological across sectors. Along the BGP, trade is balanced.

3 Investment network

A key input to the measurement of output elasticities to sectorial productivity is the investment network, which we estimate. We first describe our methodology, then the data sources and finally discuss the properties of the network across countries.

To ease the exposition and analysis, we group sectors in eight categories: four equipment types that consist of Information and Communication Technology (ICT)[12], Electronics, Machinery and Transportation Equipment, and then Construction, Agriculture, Manufacturing (other-than equipment), and Services (see Table 9 in the Appendix A.3 for details).

[12] Software is including under ICT equipment which is in turn produced by the Information and communication sector, see Table 9 for our correspondence.

3.1 Methodology

An entry (i, j) of the capital flows table records total investment expenditures by column-sector j purchased from row-sector i. Summing across columns for each row in this table generates total production of investment by each sector, while summing across rows for each column generates total investment expenditures for each sector. To obtain the investment network expressed in terms of expenditure shares, we simply divide each entry of column-sector j by total expenditures in that sector.

We classify sectors between those that produce equipment, structures (construction) and other goods. Estimates of investment produced by each sector in the economy come from *Use tables*, which record the uses of output between intermediate and final uses, including consumption and investment. Next, we estimate how much of the investment flow from each sector is purchased by any other sector in the economy. To do so, we follow two different assignment rules that depend on whether the sector produces equipment or other goods, as we describe next.

Allocation of Equipment Investment Flows. We allocate investment flows following the methodology historically implemented by the BEA. This methodology exploits the occupational composition of the labor force in each sector and the types of capital that these occupations use to assign stocks to workers. BEA's allocation is ad-hoc, as outlined in their publicly available documentation. In contrast, our assignment follows the tools utilized in each occupation, as described by O*NET, and implements the methodology introduced by Caunedo *et al.* (2023) for assigning stocks to workers in the US. We cross-walk equipment categories with the corresponding tools within each SOC occupation.[13] Therefore, our identification assumption is that the relative intensity of computer use between engineers and janitors, for instance, is the same across countries. Total investment assigned to each worker in a given occupation will differ across countries because the aggregate investment flow of computers vs. cars, for instance, is different across countries. Finally, the allocation of investment to sectors will also differ due to disparities in the occupational composition of the labor force.

Equipment producing sectors are $j = \{ICT, Electronics, Machinery, Transportation\}$. We compute the share of capital type j used by industry i as

$$share\, k_i^j = \frac{\tau_o^j n^{oi}}{\sum_o \tau_o^j n^{oi}}, \tag{5}$$

where n^{oi} is the number of workers in occupation o and industry i, and τ_o^j is the number of tools of capital type j used by a worker in occupation o at time t.

Construction and Other Sectors' Investment Flows. Given the absence of information on workers' use of investment from the construction and other sectors, we impute investment

[13] The methodology cross-walks equipment categories to these tools within each SOC occupation. Then we use Dingel and Neiman (2020)'s crosswalk between SOC and ISCO to map tools to harmonized cross-country occupational definitions.

analogously to intermediate inputs flows. We use the input-output structure and assign the flow of investment from a sector proportionally to their role as intermediate goods providers of other sectors in the economy.

3.2 Data description

Investment production by sector. We obtain investment production by sector from *Use Tables* that underlie the measurement of input-ouput structures. For Ghana and Ethiopia, we exploit *Use tables* provided by Mensah and de Vries (2023). For the remaining countries, we source this information from the WIOD. Flows are reported in nominal currency, which we deflate using the price of sectorial value added from the 10 Sector Dataset for Ethiopia and Ghana, and the price deflators available at the WIOD for the remaining countries.

Employment by occupation and sector. We use the measurement available in the Living Standards Measurement Study for Ghana, IPUMS International for Ethiopia, and PIAAC's survey for the remainder countries. We favor PIAAC over IPUMS international for the occupational composition of each sector because the level of occupational disaggregation is higher in the former than the latter.[14]

Input-output structure. We compute the share of intermediate inputs purchased by each sector in the economy, such that each row of the input-output matrix sums to one. We use this production shares to allocate the production of investment for Construction, Agriculture, Manufacturing and Services.

Country Coverage. Our benchmark dataset covers 22 countries at different stages of development, with income levels ranging from 1450 and 112229 PPP GDP per capita (PPP), and through time. See Table 8 in the Appendix for a full description.

3.3 The Investment Network in the development spectrum

We start by characterizing the *outdegree* of each sector the investment network, a measure of the relevance of each sector as an investment provider to other sectors of the economy. The *outdegree* is the row sum of the entries in the investment network.

Along the development spectrum, there is an increase in the outdegree of ICT and Transportation as economies develop, see Table 1. At the same time, the role of the service sector as a provider of capital to other sectors declines with income per capita, with high-income countries exhibiting an outdegree that is half of that in low-income countries. The sectors that report the largest final output towards investment within services include repairs of durables as well as wholesale and retail trade.

[14]See Caunedo, Keller and Shin (2021) for a comparison of the employment composition of the labor force across sources. PIAAC measurement aggregated at the 1-digit level correlates strongly with IPUMS data.

Table 1: Outdegrees: investment network

	Low Income	Medium Income	High Income
Agriculture	0.11	0.09	0.07
Construction	2.41	2.90	2.42
Electronics	0.46	0.56	0.50
ICT	0.04	0.51	1.05
Machinery	1.01	1.05	0.80
Manufacturing	0.66	0.60	0.82
Services	2.60	1.06	1.26
Transportation	0.70	1.22	1.08

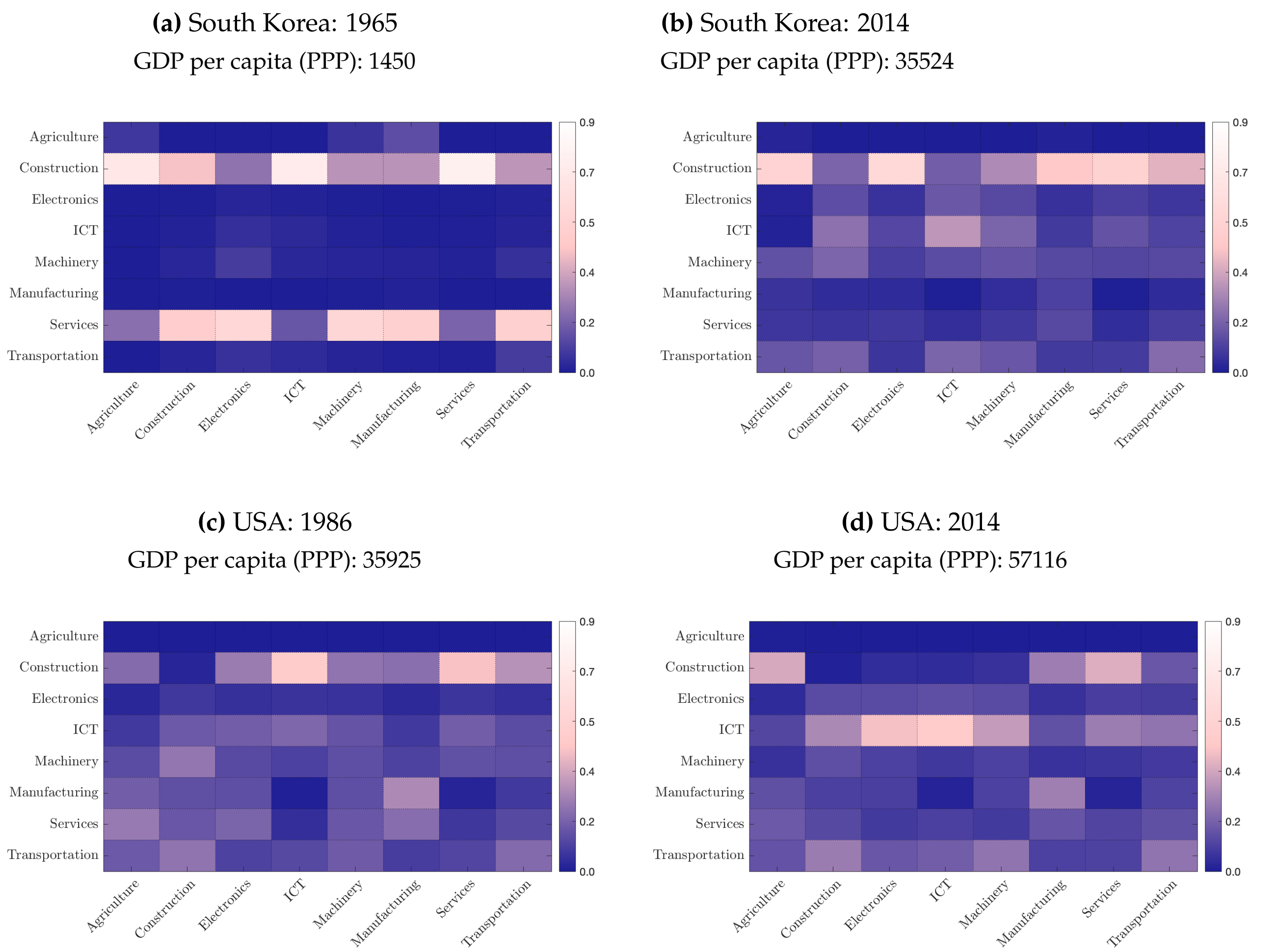
Notes: Low Income countries have an average per capita GDP (PPP) of 5030, Medium Income countries an average per capita GDP (PPP) of 44472, and High Income countries an average per capita GDP (PPP) of 84671 in 2005.

We find similar patterns when we analyze long time-series for the countries for which we have data dating back to 1960s. Figure 1 provides a graphical representation of South Korea's investment network, Ω, and shows how it changes over time. In 1965, when South Korea's GDP per capita was PPP$1450, Construction and Services sectors were star providers of investment for all sectors in the economy (Figure 1, panel a). By 2014, GDP per capita was 24.5 times higher, the role of both these sectors declines, while ICT and Transportation gain importance. Interestingly, this investment network resembles that in the USA in 1986—a period of comparable level of development (Figure 1, panel b and c)—except possibly for the role of Electronics.

If we compare de investment network of Korea at a level of GDP per capita that is 65% of the US in 2014 to the US, the most salient feature is the further raise in the importance of ICT equipment, and a decline in the importance of Construction and Machinery as providers of investment to the rest of the economy.

Differences in investment network across countries through time or income levels could prima facie reflect systematic disparities in technologies for production, either as a result of distortions or comparative advantage. The study of the sources of these disparities exceeds the scope of the current analysis but are nevertheless of key importance to understand the process of development. As a first step to highlight the implications of these newly uncovered patterns for income differences across countries, we now combine the structural predictions of the model with our newly constructed measures of the investment network to conduct an income accounting exercise.

Figure 1: Investment Network over time



Note: Investment-networks Ω for South Korea and USA. Rows label the sectors sourcing investment while Columns label the sectors receiving these flows. Each entry is the expenditure share in a particular investment relative to the total in the sector, column-sums add up to one.

4 Income Accounting

With our newly constructed measures of the investment network, we are now ready to quantify the influence vector, which summarizes output elasticities to sectorial productivity growth and to changes in the terms of trade. For the current version, we focus on productivity changes. We first describe the main sources for measurement of its component, and then conduct income accounting.

4.1 Data description

Input-output structure. We compute the share of intermediate inputs purchased by each sector in the economy, such that each row of the input-output matrix sums to one. The input-output structure is sourced from Mensah and de Vries (2023) and the WIOD.

Value added shares in production (Γ) and Sectorial value-added shares ($\tilde{\zeta}$). We compute sectoral value added shares in gross output and sectoral value added shares using data from Mensah and de Vries (2023) and the WIOD.

Capital share in value added. We exploit data from Penn World Tables version 10.01 to compute labor expenditure share. We estimate capital shares as residuals from labor expenditure shares, under the assumption of CRS value-added production technologies. The capital expenditure share is computed at the aggregate level, and therefore country-specific but common across sectors. For those countries with data from the WIOD, we use sector specific measures of capital expenditure shares (from 2000 onwards).

The sample of countries and time-horizon is the same as that of the investment network.

TFP. For each country, we estimate the model-based sectoral productivity as follows:

$$lnTFP_{n,c} = \Xi^{-1}\Gamma\, lnv_{n,c}, \tag{6}$$

with $\Xi \equiv (I - \Gamma\alpha\Omega - (1-\Gamma)M)^{-1}$, and $lnv_{n,c}$ sectoral value added in sector n and country c. To ensure comparability across countries, we rely on data from WIOD in the year 2005 to compute sectoral value added estimates, as it is the only year with available Purchasing Power Parity (PPP) sectoral value added price indices that we use to convert nominal inputs to real units. For the case of Ethiopia and Ghana, as these PPP price indices are not available, we use sectoral value added price deflators from 10 Sector Database and combine them with GDP PPP price deflators from Penn World Tables (PWT).

4.2 Accounting

We are now ready to assess the sources of cross-country income disparities accounting for the direct and indirect effects of intermediate input and investment trade across sectors. First, we compute the sum across sectors of the product between sectoral multipliers and sectoral TFP levels. To study the amplification properties of the direct and indirect effects of input-output and investment linkages, in Table 2 we compare the cross-country variance of aggregate income per capita when abstract from these links. We start with a scenario where sectorial influence only reflects the direct impact of sectoral value added; then we consider an scenario with intermediate input links only; and another scenario with the investment network only.

Without linkages, only 25% of the observed cross-country income variance can be explained (Row 4, Table 2). Differences in the input-output structure can explain 66% of the the observed income disparities (Row 3, Table 2). Hence, the remainder 34% should be attributed either to the direct effect of the investment network, or to interactions between the IO structure and the investment network. The direct impact of the investment network can explain 6% stronger amplification than the no links economy (Row 2, Table 2), so most of the role of the investment network in explaining observed income differences occurs through interactions with the intermediate inputs structure.

Table 2: Development Accounting

GDP per capita	Model Counterpart	Variance
Baseline	$(I - \Gamma\alpha\Omega - (1-\Gamma)M)^{-1}\Gamma\lambda$	0.99
Only Investment Links	$(I - \Gamma\alpha\Omega)^{-1}\Gamma\lambda$	0.31
Only Intermediate Inputs Links	$(I - (1-\Gamma)M)^{-1}\Gamma\lambda$	0.66
No Links	$\Gamma\lambda$	0.25

Counterfactuals The accounting exercises presented above abstracts from potential interactions between intermediate and investment flows. To assess those interactions we design two counterfactual scenarios in which we substitute the investment network of an economy in 2005 by the one observed in the US in 1965 and in 2014. We interpret the network in 1965 as assigning a relatively “dated" technology for production (in terms of the investment bundle in each sector), and the network in 2014 as assigning a relatively modern technology for production.

In terms of the variables on income, we find that assigning either network reduces income differences across countries. Perhaps surprisingly, assigning the 1965 network reduces income disparities the most, see Table 3 We interpret this finding as suggestive that poor countries benefit relatively more from a “dated" technology.

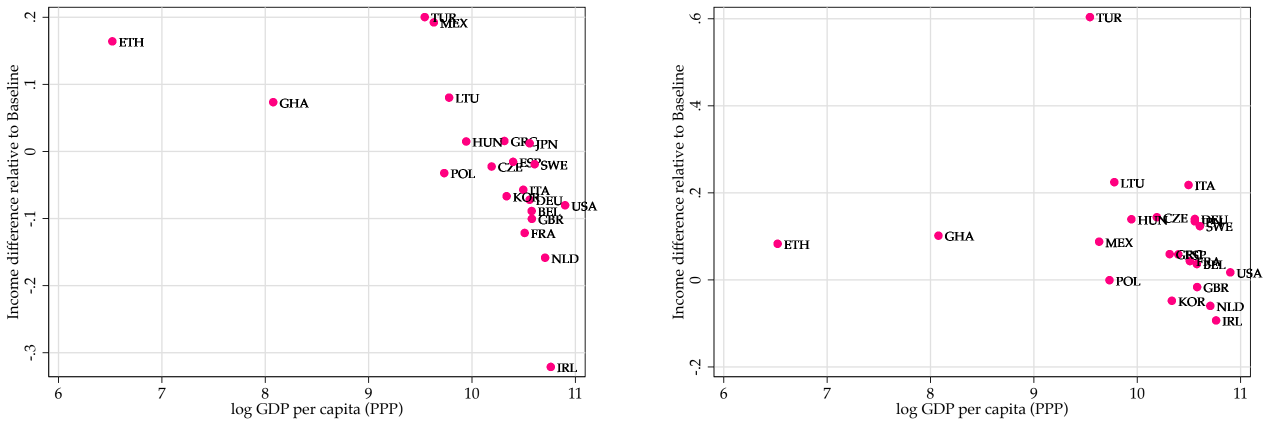
Figure 2 shows the different in income level between the model-based prediction of income per capita for each country and its counterfactual level. Hence, an outcome above 0 indicates an improvement in GDP per capita, while a negative outcome indicates a deterioration in GDP per capita. We find that richer countries would suffer a relative decline in their income per capita when producing with the investment network of US in 1965, but would benefit most from producing with the investment network of US in 2014.

Table 3: Counterfactual analysis: USA Investment Network

GDP per capita	Model Counterpart	Variance
Baseline	$(I - \Gamma\alpha\Omega - (1-\Gamma)M)^{-1}\Gamma\lambda$	0.99
USA Investment Network 2014	$(I - \Gamma\alpha\Omega_{USA2014} - (1-\Gamma)M)^{-1}\Gamma\lambda$	0.96
USA Investment Network 1965	$(I - \Gamma\alpha\Omega_{USA1965} - (1-\Gamma)M)^{-1}\Gamma\lambda$	0.85

Figure 2: Counterfactual analysis: USA Investment Network

(a) US Investment Network in 1965 **(b)** US Investment Network in 2014



4.3 The role of the Influence Vector

Disparities in amplification properties that we highlight in the income accounting exercises are summarized by the influence vector. In this section we further characterize the influence vector and compare its properties relative to the investment network and the adjusted Leontief inverse studied in vom Lehn and Winberry (2022).

Table 4 reports the magnitudes of the influence vector across countries at different stages of development. The most salient features are a steady decline in the influence of Agriculture and a steady increase in the influence of Construction, Services, ICT. Transportation, Manufacturing and Machinery (although less pronounced) display a hump-shape in the magnitude of sectorial influence. Appendix Figure 5 displays the entries in the influence vector for different equipment types across countries, whereas Appendix Figure 6 shows similar patterns for South Korea through time.

Table 4: Influence Vectors

	Low Income	Medium Income	High Income
Agriculture	0.39	0.08	0.03
Construction	0.06	0.24	0.14
Electronics	0.01	0.06	0.05
ICT	0.07	0.26	0.36
Machinery	0.03	0.08	0.05
Manufacturing	0.27	0.35	0.22
Services	0.49	0.67	0.66
Transportation	0.12	0.26	0.16

Notes: Low Income countries have an average per capita GDP (PPP) of 5030, Medium Income countries an average per capita GDP (PPP) of 44472, and High Income countries an average per capita GDP (PPP) of 84671 in 2005.

Prima facie, these patterns could be driven entirely by the sectorial shares of value added, $\tilde{\zeta}$. Hence, we separately report the outdegrees of the augmented Leontief inverse, Ξ, see Table 5. Comparing these magnitudes to those of the influence vector, it can be seen that the dynamics of influence for Services and Agriculture are mostly driven sectorial value-added shares.The reason is that the outdegrees of Leontief inverse for Services and Agriculture are relative stage along the income spectrum at levels of 3.7 and 1.4, respectively.[15] For the remainder sectors, the qualitative patterns of influence correlate with the dynamics of the outdegrees of the Leontief-inverse, although the relative magnitudes change across sectors.

[15]Korea in 1965 seems to be a bit of an outlier relative to Ghana and Ethiopia a low-income levels. Once we expand our sample to more countries, we will be able to see if this is an oddity or a feature.

Table 5: Outdegrees: adjusted-Leontief inverse

	Low Income	Medium Income	High Income
Agriculture	2.01	2.11	1.53
Construction	2.18	4.98	3.40
Electronics	1.87	3.34	2.29
ICT	1.32	3.56	5.25
Machinery	1.72	2.73	2.04
Manufacturing	7.63	10.47	6.88
Services	5.82	6.18	5.28
Transportation	2.85	4.84	3.74

Notes: Low Income countries have an average per capita GDP (PPP) of 5030, Medium Income countries an average per capita GDP (PPP) of 44472, and High Income countries an average per capita GDP (PPP) of 84671 in 2005.

One takeaway from this analysis is that the influence of Transportation, Construction and ICT increases with development, and that those are mostly driven by an increase in importance as providers of investment and intermediate inputs to the rest of the economy. To explore their role as potentially high forward-linkage sectors to the rest of the economy, we can refer again to the outdegrees of the investment network across income levels in Table 1. The outdegrees of Construction, ICT and Transportation equipment indeed increase with income levels. In other words, forward linkages from these sectors are relatively low at low-stages of development, but become more important as economies develop. Perhaps surprisingly, the role of Machinery and Manufacturing as providers of investment to the rest of the economy is relatively stable or declines with development.

So given these systematic differences in the role of sectors as providers of investment, and ultimately, in its impact on aggregate value added, we now explore their role for the observed price of capital across countries, and importantly, income disparities.

4.4 Investment and the relative price of capital

We start by replicating the well known relationship between the relative price of investment and economic development. Unfortunately, measures of the price of capital for each equipment type are not available in our sample at the correct level of disaggregation, so we work with the aggregate price of capital. Future versions of this draft will include measures of the relative price of each equipment category constructed from trade data on imported equipment.

We first correlate the sectorial influence with a measure of the aggregate relative price of capital to consumption, see Table 6. We find a strong negative correlation across all sectors producing equipment and structures, except for Transportation and Machinery (with a negative point estimate but not statistically different from zero).[16]

[16]Influence is hump-shaped in the relative price of capital.

Table 6: Relative price and influence

	Construction	Electronics	ICT	Machinery	Transportation
log_rel_price	-0.403*	-1.305***	-0.965***	-0.400	-0.0758
	(0.230)	(0.368)	(0.235)	(0.276)	(0.181)
Observations	23	23	23	23	23
R^2	0.128	0.374	0.445	0.091	0.008

Note: This sample include Korea in 1965 as a low-income country. The relative price of investment to consumption is sourced from PWT.

Table 7: Relative price and influence, controlling by income

	Construction	Electronics	ICT	Machinery	Transportation
log_rel_price	0.202	0.194	0.406**	0.241	0.271
	(0.365)	(0.494)	(0.160)	(0.448)	(0.301)
log_gdp_pc	0.302*	0.749***	0.685***	0.320*	0.173
	(0.148)	(0.200)	(0.0647)	(0.181)	(0.122)
Observations	23	23	23	23	23
R^2	0.279	0.632	0.916	0.213	0.099

Note: This sample include Korea in 1965 as a low-income country. The relative price of investment to consumption is sourced from PWT.

But it is well known that the relative price of capital correlates negatively with development, so the negative relationship to sectorial influence could be driven by disparities in income levels. Table 7 shows the correlations between our measures of influence across salient sectors and the relative price of investment to consumption, controlling for income. We find no significant relationship between these measures after accounting for income. We also find that after controlling for the price of investment to consumption, the positive relationship between influence and income per capita is only significant for Electronics and ICT. [17] For completeness we present the correlation between influence and income per capita in the Appendix.

We interpret these result to suggest that the aggregate index for the price of investment to consumption is not informative about sectorial influence beyond countries level of development. This is not to say that the price of investment is irrelevant for the observed patterns of influence. Instead, we believe that information likely relies on sector specific prices rather than the aggregate investment price. Future versions of this draft will exploit prices of imported equipment to test such a hypothesis.

[17] This result may also change as we expand our sample of countries to include more low-income economies.

5 Final Remarks

We have constructed novel measures of the investment network across the development spectrum and document systematic disparities in the importance of difference sectors as providers of investment goods as economies develop. Through a simple framework of sectorial linkages in intermediate and investment flows we show that output elasticities to sectorial productivity depend on the interaction between the input-output structure and the investment network.

Cross-country disparities in the investment network amplify the effect of sectorial disparities in TFP for income differences. We also find that setting a common investment network across countries may increase income disparities. Newer versions of this paper will include a larger cross-country coverage, mostly of poorer economies, a characterization of the speed of convergence and additional counterfactuals.

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A Appendix

A.1 Proofs & Derivations

Proof Proposition .1. Use the optimality conditions of the firm, to rewrite the expenses in different intermediate and investment goods as a function of gross output, i.e.

$$\mu_{ni}(1-\gamma_i)p_i y_i = p_n m_{ni}$$

$$\alpha_i \gamma_i p_i y_i = r_i k_i$$

$$\omega_{ji} r_i x_i = p_j \chi_{ji}$$

Combining the optimality conditions for capital and investment, as well as the steady-state level of capital

$$\alpha_i \gamma_i p_i y_i = \frac{p_j \chi_{ji}}{\omega_{ji}} \frac{k_i}{x_i},$$

which we can use to write the feasibility constraint in each sector n,

$$p_n y_n = p_n c_n + \sum_i p_n \chi_{ni} + \sum_j p_n m_{nj}.$$

Then

$$\zeta_n \frac{y_n}{c_n} = \zeta_n + \sum_i \alpha_i \gamma_i \omega_{ni} \frac{x_i}{k_i} \zeta_i \frac{y_i}{c_i} + \sum_j (1-\gamma_j) \mu_{nj} \zeta_j \frac{y_j}{c_j}$$

The above define a system of equations across sectors that can be solved for the Domar weights $\eta_n \equiv \zeta_n \frac{y_n}{c_n}$, given investment rates in each sector $\frac{x_i}{k_i}$ which proves the result.

■

Proof Proposition (open ec) .3. Use the optimality conditions of the firm, to rewrite the expenses in different intermediate and investment goods as a function of gross output, i.e.

$$\mu_{ni}(1-\gamma_i)p_i y_i = p_n m_{ni}$$

$$\alpha_i \gamma_i p_i y_i = r_i k_i$$

$$(1-\phi_j)\omega_{ji} r_i x_i = p_j \chi^d_{ji}$$

Combining the optimality conditions for capital and investment, as well as the steady-state level of capital

$$\alpha_i \gamma_i p_i y_i = \frac{p_j \chi^d_{ji}}{(1-\phi_j)\omega_{ji}} \frac{k_i}{x_i},$$

which we can use to write the feasibility constraint in each sector n,

$$p_n y_n = p_n c_n + \sum_i p_n \chi^d_{ni} + \sum_j p_n m_{nj}.$$

Then

$$\zeta_n \frac{y_n}{c_n} = \zeta_n + \sum_i \alpha_i \gamma_i (1-\phi_n) \omega_{ni} \frac{x_i}{k_i} \zeta_i \frac{y_i}{c_i} + \sum_j (1-\gamma_j) \mu_{nj} \zeta_j \frac{y_j}{c_j}$$

The above define a system of equations across sectors that can be solved for the Domar weights $\eta_n \equiv \zeta_n \frac{y_n}{c_n}$, given investment rates in each sector $\frac{x_i}{k_i}$ which proves the result.

$$\left[I - \Gamma\alpha\Omega(1-\phi)\frac{\mathbf{x}}{\mathbf{k}} - (1-\Gamma)M\right]^{-1} \boldsymbol{\zeta} \equiv \boldsymbol{\eta}$$

■

Proof Proposition .2. Use the solution and the definition of ζ_i to solve for relative prices, given investment rates.

$$\frac{p_i}{p_j} = \frac{c_j}{c_i}\frac{\zeta_i}{\zeta_j} = \frac{\eta_i}{\eta_j}\frac{y_j}{y_i}$$

These relative prices are useful to define the demand for intermediate inputs, investment and labor, as a function of the vector of sectorial gross output. The demand for intermediate inputs follows $(1-\gamma_i)\frac{\eta_i}{\eta_n} y_n = m_{ni}$, while the demand for investment goods is $\frac{x_i}{k_i}\omega_{ji}\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j = x_{ji}$. Total investment in sector i defines the level of the stock of capital as $x_i = \prod_j \left(\frac{x_i}{k_i}\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j\right)^{\omega_{ji}}$, or what is the same $k_i = \prod_j \left(\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j\right)^{\omega_{ji}}$.

Assume that the supply of labor is inelastic at 1, so the fraction of labor allocated to each sector follows Domar weights adjusted by the sectorial labor expenditure shares in gross output,

$$l_i^{\star} = \frac{(1-\alpha_i)\gamma_i p_i y_i}{\sum_i (1-\alpha_i)\gamma_i p_i y_i} = \frac{(1-\alpha_i)\gamma_i \eta_i}{\sum_i (1-\alpha_i)\gamma_i \eta_i}.$$

For the purpose of describing final demand, it would be useful to define $\tilde{l}_i = \frac{l_i^{\star}}{\gamma_i(1-\alpha_i)}$.

Final output in each sector is then

$$y_n = \left[z_n\left(\prod_i\left(\frac{\eta_n}{\eta_i}y_i\right)^{\omega_{in}}\right)^{\alpha_n}(\tilde{l}_i)^{1-\alpha_n}\right]^{\gamma_n}\left[\prod_i\left(\frac{\eta_n}{\eta_i}y_i\right)^{\mu_{in}}\right]^{1-\gamma_n}$$

Taking logs and writing output in matrix form we obtain

$$\ln(\mathbf{y}) = \Gamma\boldsymbol{\lambda} + \boldsymbol{\iota} + \Gamma\alpha\Omega'\ln(\mathbf{y}) + (1-\Gamma)M'\ln(\mathbf{y})$$

where each element of the vector $\boldsymbol{\iota}$ can be described as $\iota_n \equiv \gamma_n(1-\alpha_n)\ln(\tilde{l}_n) + \gamma_n\alpha_n\sum_i(\omega_{in})\ln(\frac{\eta_n}{\eta_i}) + (1-\gamma_n)\sum_i \mu_{in}\ln(\frac{\eta_n}{\eta_i})$. The solution for gross output is then,

$$\ln(\mathbf{y}) = \Xi\Gamma\boldsymbol{\lambda} + \Xi\boldsymbol{\iota} \tag{7}$$

where the multiplier on sectorial productivity is $\Xi \equiv (I - \Gamma\alpha_d\Omega' - (1-\Gamma)M')^{-1}$. Let the price level of the economy be normalized to $p = 1$, then aggregate value added is $\nu = \frac{p_n y_n}{\eta_n}$ for any n. We can compute a geometric average of each of the terms using the expenditure shares of consumption and investment $\tilde{\zeta}_n \equiv \zeta_n + \frac{p_n \sum_i x_{ni}}{\nu}$ as weights (since these weights add up to 1).

$$\ln(\boldsymbol{\nu}) = \sum_n \tilde{\zeta}_n \ln(p_n) + \sum_n \tilde{\zeta}_n \ln(y_n) - \sum_n \tilde{\zeta}_n \ln(\eta_n)$$

Given a CRS aggregator of sectorial output, the price index for final goods satisfies, $\ln(p) = \sum_n \zeta_n \ln(p_n)$. Because final output is the numeraire, the log of the price index equals zero, and therefore the first term in the expression for value added drops up. The weighting of the terms in the sum also include investment shares in value added. Investment shares are proportional to consumption shares in value added whenever sectorial value added shares are proportional to consumption shares across sectors. This is by construction the assumption in canonical models of input-output linkages without capital and we assume that feature here.[18]

We have already characterized the solution to each of the last two terms, in equations 4 and 8.

$$\ln(\boldsymbol{\nu}) = \tilde{\boldsymbol{\zeta}}'\Xi(\Gamma\boldsymbol{\lambda} + \boldsymbol{\iota}) - \sum_n \tilde{\zeta}_n \ln(\eta_n)$$

where we can define the elasticity of value to sectorial TFP as $\tilde{\eta} \equiv \tilde{\boldsymbol{\zeta}}'\Xi$. Unlike the Domar weight, these elasticities are not adjusted by the investment rate. At the same time, the investment rate enters into the measure of the expenditure share, $\tilde{\zeta}$ through the Domar weight because $(I - (1-\Gamma)M')^{-1}\tilde{\zeta} = \eta$.

Unpacking the vectors, $\tilde{\zeta}_j = \tilde{\eta}_j - \sum_n \gamma_n\alpha_n\omega_{jn}\tilde{\eta}_n - \sum_n (1-\gamma_n)\mu_{jn}\tilde{\eta}_n$

$$\sum_j \tilde{\zeta}_j ln(\mu_j) = \sum_j \tilde{\eta}_j \ln(\eta_j) - \sum_j\sum_i \gamma_n\alpha_n\omega_{ji}\tilde{\eta}_i \ln(\mu_j) - \sum_j\sum_i (1-\gamma_n)\mu_{ji}\tilde{\eta}_i ln(\mu_j)$$

Now consider the term, $\tilde{\eta}\iota$

$$\sum_n \tilde{\eta}_n\iota_n = \sum_n \tilde{\eta}_n\gamma_n(1-\alpha_n)\ln(\tilde{l}_n) + \tilde{\eta}_n\gamma_n\alpha_n \sum_j \omega_{jn}\ln(\frac{\eta_n}{\eta_j}) + \tilde{\eta}_n(1-\gamma_n)\sum_j \mu_{jn}\ln(\frac{\eta_n}{\eta_j}))$$

which can be rewritten as

$$\begin{aligned}\sum_n \tilde{\eta}_n\iota_n &= \sum_n \tilde{\eta}_n\gamma_n(1-\alpha_n)\ln(\tilde{l}_n) + \sum_n \tilde{\eta}_n(\gamma_n\alpha_n + 1 - \gamma_n)\ln(\eta_n) \\ &\quad - \sum_n \tilde{\eta}_n\gamma_n\alpha_n \sum_j \omega_{jn}\ln(\eta_j) - \sum_n \tilde{\eta}_n(1-\gamma_n)\sum_j \mu_{jn}\ln(\eta_j).\end{aligned}$$

Therefore the difference in the last two terms of the expression for value added are

$$\sum_n \tilde{\eta}_n\iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) = \sum_n \tilde{\eta}_n\gamma_n(1-\alpha_n)(\ln(\tilde{l}_n) - \ln(\eta_n))$$

We can rewrite this last condition solely as a function of influence vectors by replacing the optimal labor demand,

$$\sum_n \tilde{\eta}_n\iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) = \sum_n \tilde{\eta}_n\gamma_n(1-\alpha_n)(\ln(\eta_n) - \ln(\eta_n) - \ln(\sum_n \gamma_n(1-\alpha_n)\eta_n)$$

Hence,

$$\sum_n \tilde{\eta}_n\iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) = -\ln(\sum_n \gamma_n(1-\alpha_n)\eta_n)\sum_n \tilde{\eta}_n\gamma_n(1-\alpha_n)$$

which proves our result. ■

[18]Alternatively, one can set up the economy so that investment in different capital types is produced through the final good. This economy would also allow us to define the price of value added as a function of sectorial prices in a way that they drop out from the expression above, while allowing for investment shares that need not be proportional to consumption shares. The undesirable feature of this economy is that sector producing for final production and intermediate inputs are decoupled from those producing investment.

Proof Proposition .4. Use the solution and the definition of ζ_i to solve for relative prices, given investment rates.

$$\frac{p_i}{p_j} = \frac{c_j}{c_i}\frac{\zeta_i}{\zeta_j} = \frac{\eta_i}{\eta_j}\frac{y_j}{y_i}$$

These relative prices are useful to define the demand for intermediate inputs, investment and labor, as a function of the vector of sectorial gross output. The demand for intermediate inputs follows $(1-\gamma_i)\frac{\eta_i}{\eta_n}y_n = m_{ni}$, while the demand for domestic investment goods is $\frac{x_i}{k_i}(1-\phi_j)\omega_{ji}\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j = \chi_{ji}$. The demand for imported investment satisfies $\frac{x_i}{k_i}(\phi_j)\omega_{ji}\alpha_i\gamma_i\frac{\eta_i}{p_j^f}\nu = \chi_{ji}^f$.

Total investment in sector i defines the level of the stock of capital as

$$x_i = \prod_j \left(\left(\frac{x_i}{k_i}\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j\right)^{1-\phi_j} \left(\frac{x_i}{k_i}\alpha_i\gamma_i\frac{\eta_i}{p_j^f}\nu\right)^{\phi_j}\right)^{\omega_{ji}},$$

or what is the same $k_i = \prod_j \left(\left(\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j\right)^{1-\phi_j} \left(\alpha_i\gamma_i\frac{\eta_i}{p_j^f}\nu\right)^{\phi_j}\right)^{\omega_{ji}}$.

Assume that the supply of labor is inelastic at 1, so the fraction of labor allocated to each sector follows Domar weights adjusted by the sectorial labor expenditure shares in gross output,

$$l_i^\star = \frac{(1-\alpha_i)\gamma_i p_i y_i}{\sum_i (1-\alpha_i)\gamma_i p_i y_i} = \frac{(1-\alpha_i)\gamma_i\eta_i}{\sum_i(1-\alpha_i)\gamma_i\eta_i}.$$

For the purpose of describing final demand, it would be useful to define $\tilde{l}_i = \frac{l_i^\star}{\gamma_i(1-\alpha_i)}$.

Final output in each sector is then

$$y_n = \left[z_n \left(\prod_i \left((\frac{\eta_n}{\eta_i}y_i)^{1-\phi_i}(\frac{\eta_n}{p_i^f}\nu)^{\phi_i}\right)^{\omega_{in}}\right)^{\alpha_n}(\tilde{l}_i)^{1-\alpha_n}\right]^{\gamma_n}\left[\prod_i \left(\frac{\eta_n}{\eta_i}y_i\right)^{\mu_{in}}\right]^{1-\gamma_n}$$

Taking logs and writing output in matrix form we obtain

$$\ln(\mathbf{y}) = \Gamma\boldsymbol{\lambda} + \boldsymbol{\iota} + \Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}'\nu + \Gamma\alpha(1-\boldsymbol{\phi})'\Omega'\ln(\mathbf{y}) + (1-\Gamma)M'\ln(\mathbf{y})$$

where each element of the vector $\boldsymbol{\iota}$ can be described as $\iota_n \equiv \gamma_n(1-\alpha_n)\ln(\tilde{l}_n) + \gamma_n\alpha_n\sum_i(1-\phi_i)\omega_{in}\ln(\frac{\eta_n}{\eta_i}) + \gamma_n\alpha_n\sum_i\phi_i\omega_{in}\ln(\frac{\eta_n}{p_i^f}) + (1-\gamma_n)\sum_i\mu_{in}\ln(\frac{\eta_n}{\eta_i})$. The solution for gross output is then,

$$\ln(\mathbf{y}) = \Xi\Gamma\boldsymbol{\lambda} + \Xi\boldsymbol{\iota} + \Xi\Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}'\nu \tag{8}$$

where the multiplier on sectorial productivity is $\Xi \equiv (I - \Gamma\alpha\boldsymbol{\phi}'\Omega' - (1-\Gamma)M')^{-1}$. Let the price level of the economy be normalized to $p=1$, then aggregate value added is $\nu = \frac{p_n y_n}{\eta_n}$ for any n. We can compute a geometric average of each of the terms using the expenditure shares consumption and investment $\tilde{\zeta}_n \equiv \zeta_n + \frac{p_n x_n}{\nu}$ as weights (since these weights add up to 1 and trade is balanced),

$$\ln(\boldsymbol{\nu}) = \sum_n \tilde{\zeta}_n\ln(p_n) + \sum_n\tilde{\zeta}_n\ln(y_n) - \sum_n\tilde{\zeta}_n\ln(\eta_n).$$

Given a CRS aggregator of sectorial output, the price index for final goods satisfies, $\ln(p) = \sum_n \zeta_n \ln(p_n)$. Because final output is the numeraire, the log of the price index equals zero, and therefore the first term in the expression for value added drops up. The weighting of the terms in the sum also include investment shares in value added. Investment shares are proportional to consumption shares in value added whenever sectorial value added shares are proportional to consumption shares across sectors. This is by construction the assumption in canonical models of input-output linkages without capital and we assume that feature here.[19]

We have already characterized the solution to each of the last two terms, in equations 4 and 8.

$$\ln(\nu) = \tilde{\boldsymbol{\zeta}}'\Xi(\Gamma\boldsymbol{\lambda} + \boldsymbol{\iota} + \Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}'\nu) - \sum_n \tilde{\zeta}_n \ln(\eta_n)$$

where we can define the elasticity of value to sectorial TFP as $\tilde{\eta} \equiv \tilde{\boldsymbol{\zeta}}'\Xi$. Unlike the Domar weight, these elasticities are not adjusted by the investment rate. At the same time, the investment rate enters into the measure of the expenditure share, $\tilde{\zeta}$ through the Domar weight because $(I - (1-\Gamma)M')^{-1}\tilde{\zeta} = \eta$..

Because of the presence of tradable investment goods we obtain an additional amplification (as in Jones (2011) for tradable intermediate inputs). The reason is that as productivity increases within the economy, the export capacity improves, and due to trade balance that implies higher imports of investment. The strongest the dependence on imported equipment and the intensity of use of capital, the strongest is this amplification channel.

$$\ln(\nu) = \left(I - \Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}'\right)^{-1}\left[\tilde{\boldsymbol{\zeta}}'\Xi(\Gamma\boldsymbol{\lambda} + \boldsymbol{\iota}) - \sum_n \tilde{\zeta}_n \ln(\eta_n)\right]$$

Unpacking the vectors, $\tilde{\zeta}_n = \tilde{\eta}_n - \sum_j \gamma_j\alpha_j(1-\phi_j)\omega_{nj}\tilde{\eta}_j - \sum_j(1-\gamma_j)\mu_{nj}\tilde{\eta}_j$

$$\sum_n \tilde{\zeta}_n ln(\mu_n) = \sum_n \tilde{\eta}_n \ln(\eta_n) - \sum_n\sum_j \gamma_n\alpha_n(1-\phi_n)\omega_{nj}\tilde{\eta}_j \ln(\mu_n) - \sum_n\sum_j (1-\gamma_n)\mu_{nj}\tilde{\eta}_j \ln(\mu_n)$$

Now consider the term, $\tilde{\eta}\iota$

$$\begin{aligned}\sum_n \tilde{\eta}_n\iota_n &= \sum_n(\tilde{\eta}_n\gamma_n(1-\alpha_n)\ln(\tilde{l}_n) + \tilde{\eta}_n\gamma_n\alpha_n\sum_j(1-\phi_j)\omega_{jn}\ln(\frac{\eta_n}{\eta_j}) + \gamma_n\alpha_n\sum_j\phi_j\omega_{jn}\ln(\frac{\eta_n}{p_j^f}) \\ &\quad + \tilde{\eta}_n(1-\gamma_n)\sum_j\mu_{jn}\ln(\frac{\eta_n}{\eta_j}))\end{aligned}$$

which can be rewritten as

$$\sum_n \tilde{\eta}_n\iota_n = \sum_n \tilde{\eta}_n\gamma_n(1-\alpha_n)\ln(\tilde{l}_n) + \sum_n \tilde{\eta}_n(\gamma_n\alpha_n + 1 - \gamma_n)\ln(\eta_n)$$

[19] Alternatively, one can set up the economy so that investment in different capital types is produced through the final good. This economy would also allow us to define the price of value added as a function of sectorial prices in a way that they drop out from the expression above, while allowing for investment shares that need not be proportional to consumption shares. The undesirable feature of this economy is that sector producing for final production and intermediate inputs are decoupled from those producing investment.

$$-\sum_n \tilde{\eta}_n \gamma_n \alpha_n \sum_j \omega_{jn} \left((1-\phi_j)\ln(\eta_j) + \phi_j \ln(p_j^f) \right) - \sum_n \tilde{\eta}_n (1-\gamma_n) \sum_j \mu_{jn} \ln(\eta_j)$$

Therefore the difference in the last two terms of the expression for value added are

$$\sum_n \tilde{\eta}_n \iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) \quad = \quad \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n)(\ln(\tilde{l_n}) - \ln(\eta_n)) - \sum_n \tilde{\eta}_n \gamma_n \alpha_n \sum_j \omega_{jn} \phi_j \ln(p_j^f)$$

The last term can be written as a function of the terms of trade for imported equipment j, $\ln(\tau_j) = \ln(p) - \ln(p_j^f)$. Because the final good is the numeraire, p=1. Hence,

$$\sum_n \tilde{\eta}_n \iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) \quad = \quad \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n)(\ln(\tilde{l_n}) - \ln(\eta_n)) + \sum_n \tilde{\eta}_n \gamma_n \alpha_n \sum_j \omega_{jn} \phi_j \ln(\tau_j)$$

which proves our result. ■

A.1.1 Balanced growth path

Let us start by defining GDP in the economy, ν as the value of consumption and investment expenses plus net exports, $C + \sum p_n x_n + NX = \nu$, in units of consumption.

Definition: *A balanced growth path is an allocation where output, consumption, investment and capital in each sector grow at a constant, possibly different, growth rate.*

Along the BGP

$$g^\nu = g^c = g^{p^x} + g^x = g^{NX},$$

The growth rate of net exports is

$$g^{NX} = g^\epsilon = g^{p^f} + g^{\chi^f}.$$

It follows that the growth rate of the terms of trade (considered exogenous) determines the relative growth of real exports and imports whenever trade is balanced.

$$g^\tau \equiv -g^{p^f} = g^{\chi^f} - g^\epsilon. \tag{9}$$

Define g^y as the vector collecting the growth rates of gross output across sectors $g^y = (g^{y_1}, \ldots.., g^{y_N})$. We define g^ν, g^m, g^k and g^x analogously. The growth rate of output in each sector grows at a constant rate equal to growth rate of its uses, including consumption, investment and intermediate goods. Feasibility then implies that $g^{m_{in}} = g^{y_i}$, and therefore, given the aggregator of intermediate inputs in sector n, $g^{m_n} = \sum_{i=0}^N \mu_{in} g^{y_i}$. In other words, $g^{m_n} = M' g^y$.

Along the BGP, the law of motion for capital requires $g^x = g^k$, where investment includes domestically and foreign sourced investment. Hence,

$$g^k = g^x = (1-\phi)\Omega' g^{\chi^d} + \phi \Omega' g^{\chi^f}$$

$$g^k = g^x = (1-\phi)\Omega' g^{\chi^d} + \phi \Omega' g^\epsilon + \phi \Omega' g^\tau$$

Note that because of trade balance the amount of exports in equilibrium equals the amount of imported equipment.

Finally, the production technology implies $g^y = \Gamma g^\nu + (1-\Gamma)g^m$, and by definition, $g^\nu = g^z + \alpha g^k + (1-\alpha)g^l$. But aggregate labor supply is fixed and along a BGP the share of labor allocated to each sector is constant (because relative sectorial output is constant). Using the growth rate of capital and collecting the terms with the growth rate of gross output yields $g^y = \Gamma g^z + \Gamma\alpha(1-\phi)\Omega' g^y + \Gamma\alpha\phi\Omega' g^\nu + \Gamma\alpha\phi\Omega' g^\tau) + (1-\Gamma)M' g^y$. The third term in the RHS of this expression corresponds to the growth rate of exports.

$$g^y = \Xi'\Gamma(g^z + \alpha\phi\Omega' g^{\underline{\tau}} + \alpha\phi\Omega' g^\nu).$$

Hence, the growth rate of gross output in the economy depends on the productivity growth in each sector, the terms of trade and the growth rate of exports, proportional to value added, with a multiplier $\Xi' \equiv (I - \Gamma\alpha(1-\phi)\Omega' - (1-\Gamma)M')^{-1}$. The matrix Ξ is the generalized Leontief inverse.

$$g^\nu = (I - \alpha\phi\Omega'(I + \alpha(1-\phi)\Omega'\Xi'\Gamma))^{-1}(I + \alpha(1-\phi)\Omega'\Xi'\Gamma)[g^z + \alpha\phi g^\tau]$$

Hence, the growth rate of value added follows from the factor structure of the economy, as in Long and Plosser (1983).

A.2 Technology choices

In this section we micro-found disparities in the investment aggregator along the development spectrum. We construct an economy where firms choose investment demand in each sector, as well as investment intensity across different capital types. Our benchmark economy is a special case of this general problem.

We populate the economy by a continuum of firms that produce investment goods for each sector. These firms maximize profits by choosing the amount of investment in each equipment type, but also the intensity of use of each equipment for production following

$$\max_{\omega_{int}, \chi_{int}} r_{nt} x_{nt} - \sum_i p_{it} \chi_{int}$$

subject to

$$x_{nt} = \sum_{i=1}^{N} \left(\omega_{int} \chi_{int}^{\sigma_n}\right)^{\frac{1}{\sigma_n}}, \tag{10}$$

$$\sum_i \xi_{int} \omega_{int}^{\nu_n} = B_n \tag{11}$$

The production technology is a generalization of the investment aggregator described in equation 1. The technology frontier, is a generalization of Caselli and Coleman (2006) to allow for multiple equipment types, and sector-specific technology barriers, B_n. As we will show next, a key difference to their environment is that firms choose expenses in goods that are produced within the economy, rather than goods that the economy is endowed with (i.e. labor in their case). Hence, there is a non-trivial feedback between the nature of the investment network and IO structure, which determine relative prices, and technology choice.

The optimal (interior) choices of firms are characterized by two conditions

$$\left(\frac{\chi_{jnt}}{\chi_{int}}\right)^{1-\sigma_n} = \frac{\omega_{jnt}}{\omega_{int}} \frac{p_{it}}{p_{jt}} \tag{12}$$

$$\left(\frac{\chi_{int}}{\chi_{jnt}}\right)^{\sigma_n} = \frac{\xi_{int}}{\xi_{jnt}} \left(\frac{\omega_{int}}{\omega_{jnt}}\right)^{\nu_n - 1} \tag{13}$$

Replacing 12 into 13 we obtain

$$\frac{\chi_{int}}{\chi_{jnt}} = \left(\frac{\xi_{int}}{\xi_{jnt}} \left(\frac{p_{it}}{p_{jt}}\right)^{\nu_n - 1}\right)^{\frac{1}{\sigma_n \nu_n + 1 - \nu_n}} \tag{14}$$

as well as

$$\frac{\omega_{jnt}}{\omega_{int}} = \left(\frac{\xi_{jnt}}{\xi_{int}}\right)^{\frac{1-\sigma_n}{\sigma_n \nu_n + 1 - \nu_n}} \left(\frac{p_{jt}}{p_{it}}\right)^{\frac{\sigma_n}{\sigma_n \nu_n + 1 - \nu_n}} \tag{15}$$

Hence, if $\sigma_n \nu_n - (\nu_n - 1) < 0$ we obtain an interior solution. This is the same as requiring, $\nu_n > 1/(1 - \sigma_n)$. Such a condition requires more curvature in the technology choice than in the investment aggregator. As in Caselli and Coleman (2006), if $\sigma_n < 0$ firms choose to increase the efficiency of the relatively expensive factor, while if $\sigma_n > 0$, they increase the efficiency of the relatively cheap factor. At the same time, the relative demand for a particular investment type decreases in its price.

This economy reduces to our benchmark economy as we take the limit when $\sigma_n \to 0$. In that case, expenditure shares in the investment aggregators are simply the parameters characterizing the shape of the production possibility frontier in each economy $\omega_{int} \propto \xi_{int}^{\frac{1}{1-\nu_n}}$, and independent of relative prices.

In other words, an interpretation of our counterfactual analysis that assigns the investment network of an economy to another one is that we assign different production possibility frontiers to these economies. The level of the frontier B_n is identified by the level difference in the expenditure share for a given category in a given sector; while the shape of the frontier is identified from the distribution of expenditure shares in a given sector across equipment types.

A.3 Data appendix

Table 8: Country Sample and Data Sources

	Country	GDP per capita (PPP)	Use-Tables; Input-Output Matrix Value Added Share in Gross Output		Employment by Occupation and Industry	
			Source	*Available Years*	*Source*	*Available Years*
1	Ethiopia	679	MDV	1990-2019	IPUMS	1994
2	Rwanda	1246	MDV	1990-2019	IPUMS	2002,2012
3	Tanzania	1507	MDV	1990-2019	IPUMS	2002,2012
4	Zambia	1710	MDV	1990-2019	IPUMS	1990,2000,2010
5	Kenya	1972	MDV	1990-2019	KEN HH survey	2021
6	Cambodia	2048	OECD	2005-2015	IPUMS	1998,2004,2008,2013
7	Senegal	2728	MDV	1990-2019	IPUMS	1988,2013
8	Cameroon	2743	MDV	1990-2019	IPUMS	2005
9	India	2872	WIOD	1965-2000; 2000-2014	IPUMS	1983,1987,1993,1999,2004,2009
10	Vietnam	3128	OECD	2005-2015	IPUMS	1989,1999,2009
11	Ghana	3219	MDV	1990-2019	IPUMS, LSMS	1984,2000,2010;2009
12	Nigeria	3481	MDV	1990-2019	IPUMS	2006-2010
13	Philippines	4366	OECD	2005-2015	IPUMS	1990,1995,2000,2010
14	Indonesia	4602	OECD	2005-2015	IPUMS	1971,1976,1980,1985,1990,1995,2000,2005,2010
15	Morocco	4672	OECD	2005-2015	IPUMS	1982,1994,2004,2014
16	China	6681	OECD	2005-2015	IPUMS	1982,1990,2000
17	Peru	6832	OECD	2005-2015	IPUMS	1993,2007
18	Colombia	8367	OECD	2005-2015	IPUMS	1964,1973,1993,2005
19	Tunisia	9353	OECD	2005-2015	ILOSTAT	
20	Brazil	9610	WIOD	1965-2000; 2000-2014	IPUMS	1960,1970,1980,1991,2000,2010
21	Thailand	10293	OECD	2005-2015	IPUMS	1970,1980,1990,2000
22	Kazakhstan	11176	OECD	2005-2015		
23	Romania	11186	OECD	2005-2015	IPUMS	1977,1992,2002,2011
24	South Africa	11311	MDV,OECD	1990-2019; 2005-2015	IPUMS	1996,2001-2007
25	Costa Rica	11580	OECD	2005-2015	IPUMS	1963,1973,1984,2000,2011
26	Bulgaria	11919	OECD	2005-2015		
27	Turkey	13941	WIOD	2000-2014	IPUMS, PIAAC	1985,1990,2000; 2015
28	Argentina	14247	OECD	2005-2015	IPUMS	1970,1980,1991,2001
29	Mauritius	14325	MDV	1990-2019	IPUMS	1990,2000,2011
30	Chile	14534	OECD	2005-2015	IPUMS	1960,1970,1982,1992,2002,2017
31	Mexico	15230	WIOD	1965-2000; 2000-2014	IPUMS,PIAAC	1960,1970,1990,1995, 2005-2019;2017

	Country	GDP per capita (PPP)	Use-Tables; Input-Output Matrix Value Added Share in Gross Output		Employment by Occupation and Industry	
			Source	*Available Years*	*Source*	*Available Years*
32	Russia	15450	WIOD	2000-2014	PIAAC	2012
33	Latvia	16144	WIOD	2000-2014		
34	Poland	16838	WIOD	2000-2014	IPUSM,PIAAC	1978,2002;2012
35	Malaysia	17412	OECD	2005-2015	IPUMS	1970,1980-1991,2000
36	Lithuania	17646	WIOD	2000-2014	PIAAC	2015
37	Croatia	18029	WIOD	2000-2014		
38	Estonia	19819	WIOD	2000-2014	ILOSTAT	
39	Slovakia	20168	WIOD	2000-2014	ILOSTAT	
40	Hungary	20819	WIOD	2000-2014	IPUMS,PIAAC	2001,2011;2017
41	Malta	26536	WIOD	2000-2014	ILOSTAT	
42	Czechia	26624	WIOD	2000-2014	PIAAC	2012
43	Portugal	27149	WIOD	1965-2000; 2000-2014	IPUMS	1981,1991-2001,2011
44	Slovenia	28821	WIOD	2000-2014	ILSOTAT	
45	Greece	30138	WIOD	1965-2000; 2000-2014	PIAAC	2015
46	South Korea	30784	WIOD	1965-2000; 2000-2014	PIAAC	2012
47	Saudi Arabia	31380	OECD	2005-2015		
48	New Zealand	31485	OECD	2005-2015		
49	Israel	32358	OECD	2005-2015	IPUMS	1972,1983,1995,2008
50	Spain	32769	WIOD	1965-2000; 2000-2014	IPUMS,PIAAC	1981,1991,2001,2005-2020;2012
51	Cyprus	33025	WIOD	2000-2014	ILOSTAT	
52	Italy	36167	WIOD	1965-2000; 2000-2014	IPUMS,PIAAC	2001,2011-2020;2012
53	France	36651	WIOD	1965-2000; 2000-2014	IPUMS,PIAAC	1962-1968,1975-1982,1990,1999,2006,2011;2012
54	Finland	38435	WIOD	1965-2000; 2000-2014	IPUMS	2010
55	Japan	38466	WIOD	1965-2000; 2000-2014	PIAAC	2012
56	Germany	38475	WIOD	1965-2000; 2000-2014	IPUMS,PIAAC	1970,1971,1981,1987;2012
57	Taiwan	39031	WIOD	1965-2000; 2000-2014		
58	Belgium	39220	WIOD	1965-2000; 2000-2014	PIAAC	2012

	Country	GDP per capita (PPP)	Use-Tables; Input-Output Matrix Value Added Share in Gross Output		Employment by Occupation and Industry	
			Source	*Available Years*	*Source*	*Available Years*
59	United Kingdom	39308	WIOD	1965-2000; 2000-2014	IPUMS,PIAAC	1991,2001;2012
60	Denmark	40344	WIOD	1965-2000; 2000-2014		
61	Sweden	40381	WIOD	1965-2000; 2000-2014	PIAAC	2012
62	Austria	41678	WIOD	1965-2000; 2000-2014	IPUMS	1971-2001,2011
63	Australia	43333	WIOD	1965-2000; 2000-2014	ILOSTAT	
64	Netherlands	44662	WIOD	1965-2000; 2000-2014	IPUMS,PIAAC	1960,1971,2001,2011; 2012
65	Canada	44671	WIOD	1965-2000; 2000-2014	IPUMS	1971,1981,1991-2001, 2011
66	Ireland	47211	WIOD	1965-2000; 2000-2014	IPUMS,PIAAC	1971,1981,1986,1991,1996,2002, 2006,2011,2016;2012
67	China, Hong Kong SAR	47716	WIOD, OECD	1965-2000; 2005-2015	ILOSTAT	
68	Iceland	48377	OECD	2005-2015		
69	Switzerland	49859	OECD	2005-2015	IPUMS	1970-2000
70	Norway	54200	WIOD	2000-2014		
71	United States	54210	WIOD	1965-2000; 2000-2014	IPUMS,PIAAC (check IPUMS US)	1960,1970,1980,1990,2000-2005, 2010-2015;2017
72	Singapore	63949	OECD	2005-2015	ILOSTAT	
74	Brunei Darussalam	78708	OECD	2005-2015	ILOSTAT	
75	Luxembourg	815201	WIOD	2000-2014		

Table 9: Aggregate Sectors Definition

GGDC Sector	GGDC Sector Description	Aggregate Sector
A	Agriculture, forestry and fishing	Agriculture
C10t12	Manufacture of food products and beverages, tobacco products	Manufacturing
C13t15	Manufacture of textiles; wearing apparel; leather and related products	Manufacturing
C16t18	Manufacture of wood and of products, except furniture; articles of straw and plaiting materials; paper and paper products; Printing and reproduction of recorded media	Manufacturing
C19t22	Manufacture of coke and refined petroleum products; chemicals and chemical products; basic pharmaceutical products and pharmaceutical preparations; rubber and plastics products	Manufacturing
C23t25	Manufacture of other non-metallic mineral products; basic metals; fabricated metal products, except machinery and equipment	Manufacturing
C26t27	Manufacture of computer, electronic and optical products; electrical equipment	Electronics
C28	Manufacture of machinery and equipment n.e.c.	Machinery
C29t30	Manufacture of motor vehicles, trailers and semi-trailers; other transport equipment; Transportation and storage services	Transportation
C31t33	Manufacture of furniture; Other manufacturing; Repair and installation of machinery and equipment	Machinery
DtE	Electricity, gas, steam and air conditioning supply; Water supply; sewerage, waste management and remediation activities	Services
F	Construction	Construction
GnI	Wholesale and retail trade; repair of motor vehicles and motorcycles; Accommodation and food service activities	Services
JnMN	Information and communication; Professional, scientific and technical activities; Administrative and support service activities	ICT
K	Financial and insurance activities	Services
L	Real estate activities	Services
OtQ	Public administration and defence; compulsory social security; Education; Human health and social work activities	Services
RtU	Arts, entertainment and recreation; Other service activities; Activities of households as employers; goods- and services-producing activities of households for own use; Activities of extraterritorial organizations and bodies	Services

B Additional Tables & Pictures

Figure 3: TFP Component Contribution vs Observed GDP per capita

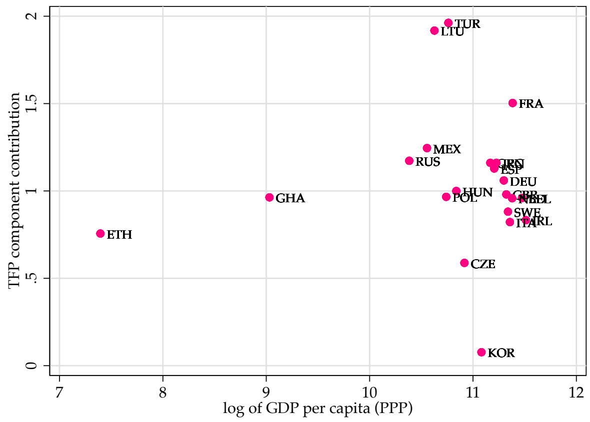
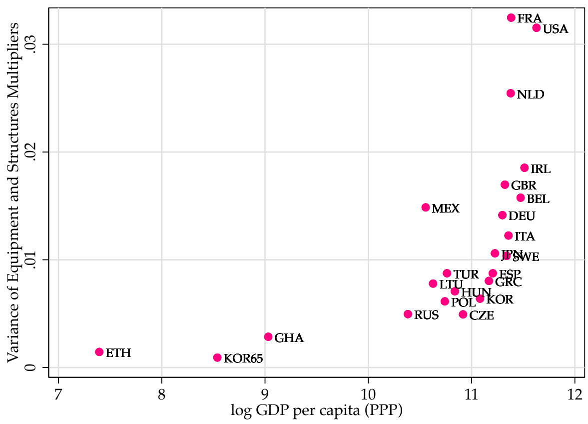
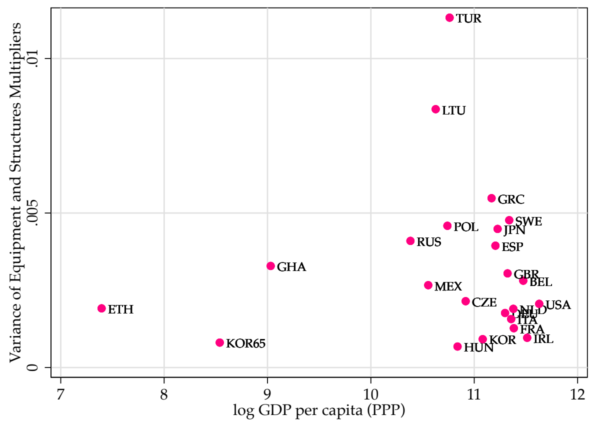


Figure 4: Variance of (...) with and without ICT

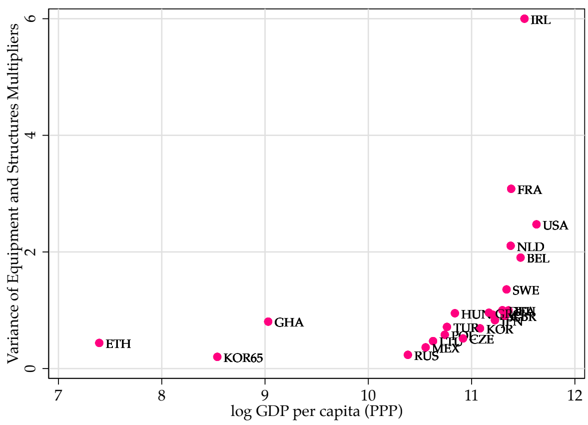
(a) Influence vector



(b) Influence vector, w/o ICT



(c) Outdegree of Augmented Leontief Inverse



(d) Outdegree of Augmented Leontief Inverse, w/o ICT

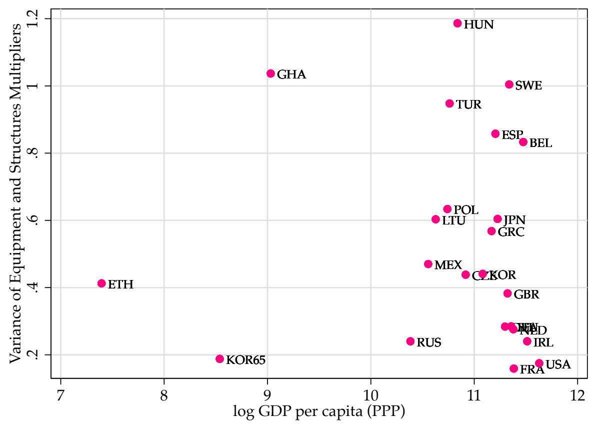


Figure 5: Influence and GDP per capita – Cross Country, 2005

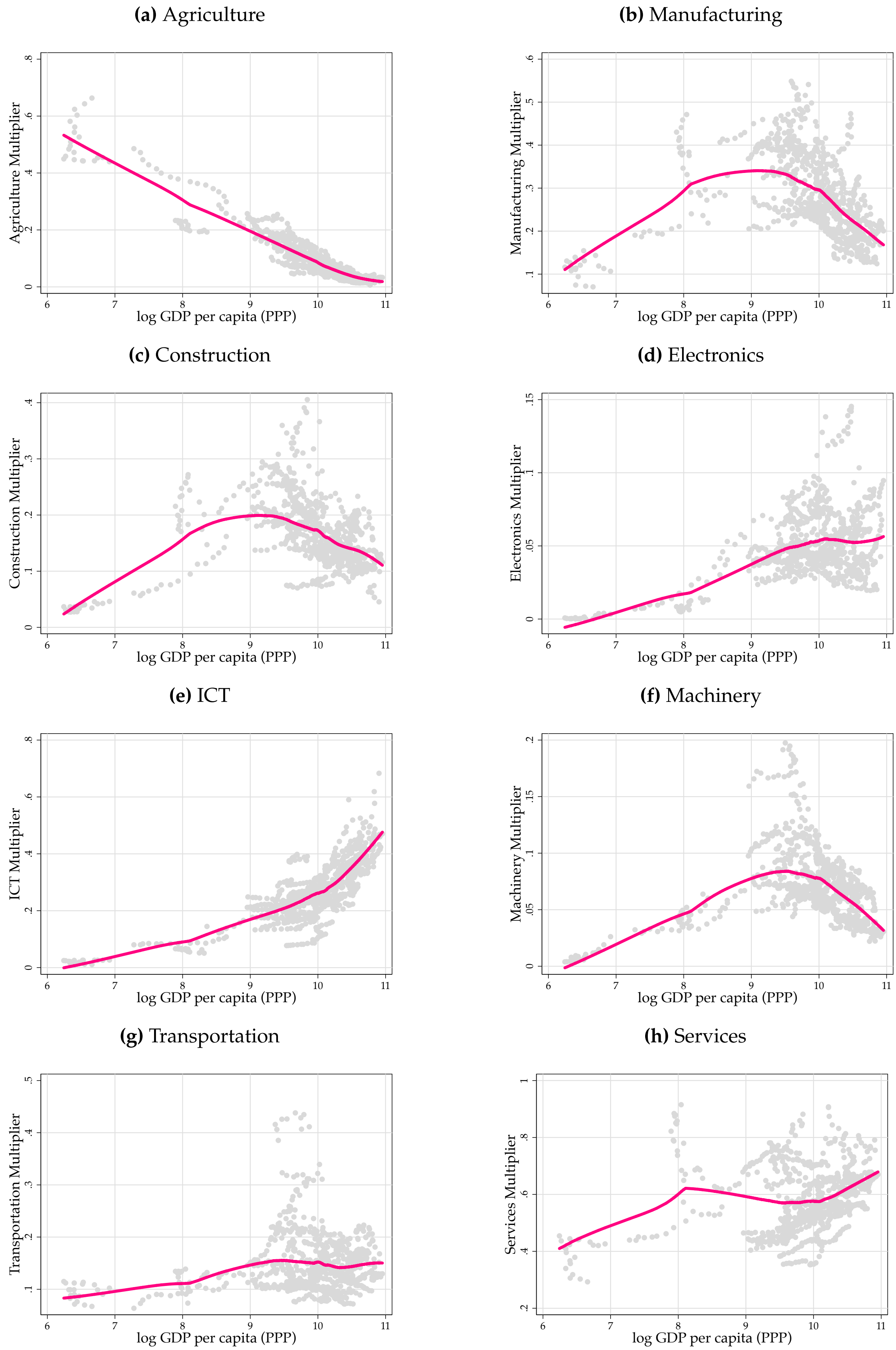
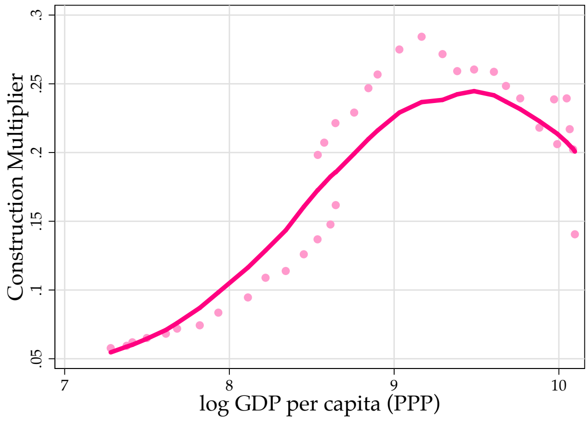
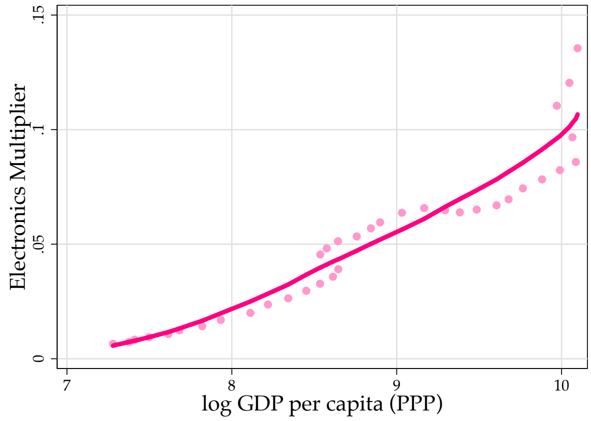


Figure 6: Influence and GDP per capita – South Korea, 1965-2000

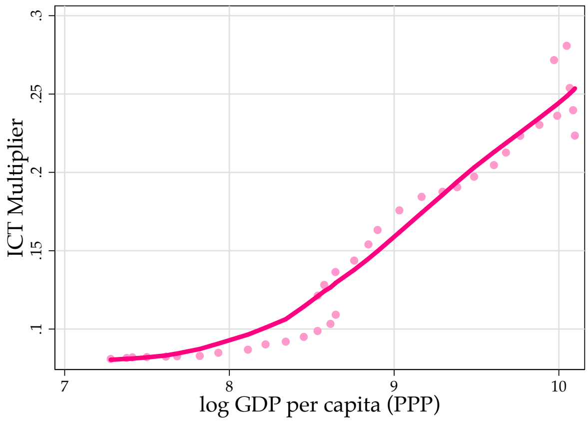
(a) Construction



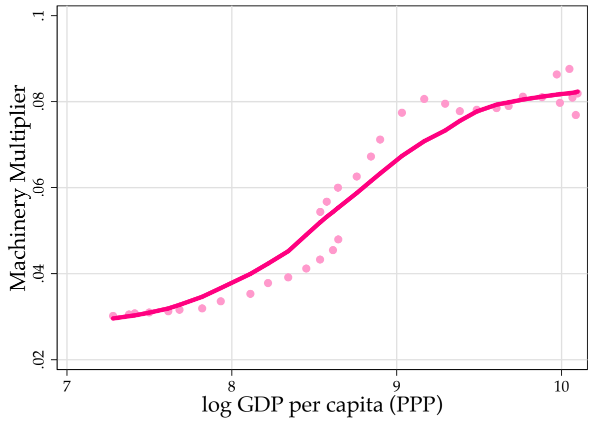
(b) Electronics



(c) ICT



(d) Machinery



(e) Transportation

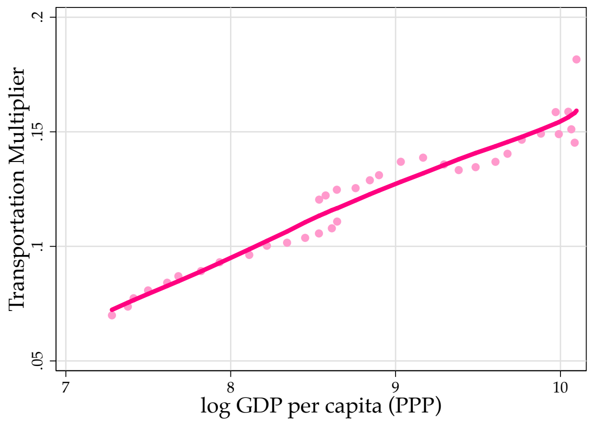
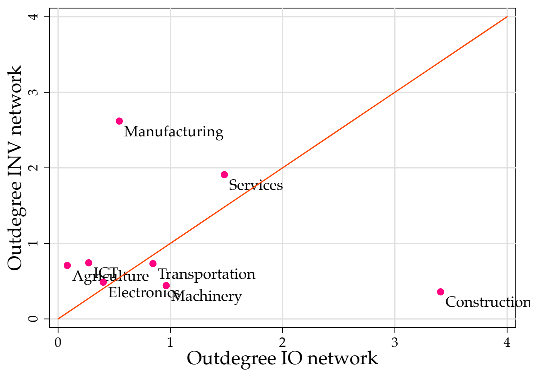
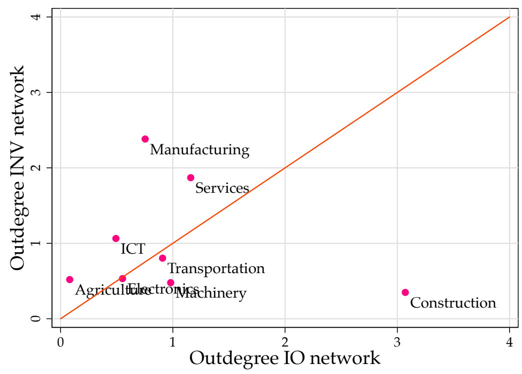


Figure 7: Outdegrees: Investment vs Production Network

(a) Low Income



(b) Medium Income



(c) High Income

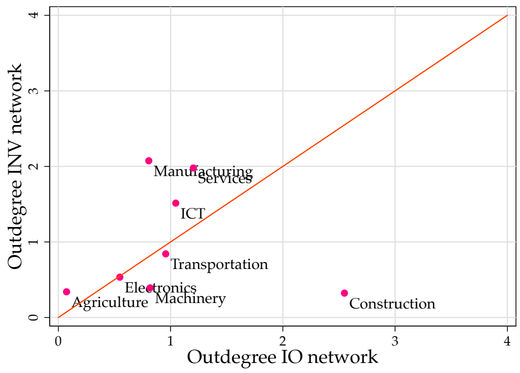
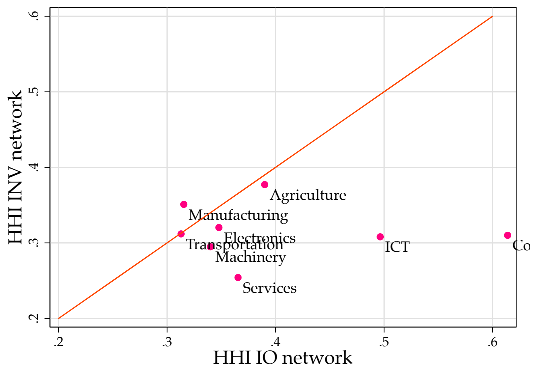
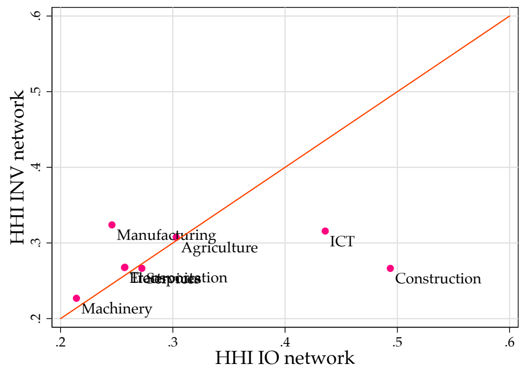


Figure 8: Sectoral Herfindahl Index: Investment vs Production Network

(a) Low Income



(b) Medium Income



(c) High Income

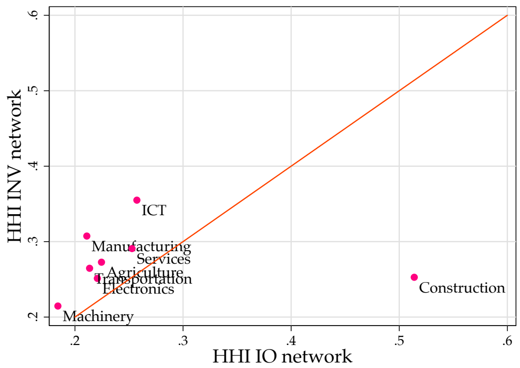


Table 10: GDP per capita and influence

	Construction	Electronics	ICT	Machinery	Transportation
log_gdp_pc	0.236**	0.685***	0.552***	0.241**	0.0844
	(0.0852)	(0.115)	(0.0426)	(0.105)	(0.0710)
Observations	23	23	23	23	23
R^2	0.268	0.629	0.889	0.202	0.063